6.5 A reversible framework for propositional bases merging

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Abstract

The problem of merging multiple sources information is central in several domains of computer science. In knowledge representation for artificial intelligence, several approaches have been proposed for merging propositional bases. However none of these approaches allows us the reversibility of the merging process. In this paper, we propose a very general reversible framework for merging ordered as well as not ordered pieces of information coming from various sources either ordered or not. A semantic approach of merging in the proposed reversible framework is first presented, stemming from a representation of total pre-orders by means of polynomials on real numbers. The syntactic counter-part is then presented, based on belief bases weighted by polynomials on real numbers. We show the equivalence between semantic and syntactic approaches. Finally, we show how this reversible framework is suitable for easily representing the approach of merging propositional bases stemming from Hamming distance and how the proposed framework is suitable for generalizing the revision of an epistemic state by an epistemic state to the fusion of several epistemic states.

Introduction

Merging information coming from different sources is an important issue in various domains of computer science like knowledge representation for artificial intelligence, decision making or databases. The aim of fusion is to obtain a global point of view, exploiting the complementarity between sources, solving different existing conflicts, reducing the possible redundancies. Among the various approaches of multiple sources information merging, logical approaches gave rise to increasing interest in the last decade (Baral *et al.* 1992; Revesz 1993; Lin 1996; Revesz 1997; Cholvy 1998). Most of these approaches have been defined within the framework of classical logic, more often propositional, and have been semantically defined. Different postulates characterizing the rational behaviour of fusion operators have been proposed (Konieczny & Pérez 1998) and various operators have been defined according to whether explicit or implicit priorities are available (Konieczny & Pérez 1998), (Laffage & Lang 2000). More recently, new approaches have been proposed like semantic merging for propositional bases stemming from the Hamming distance (Konieczny, Lang, & Marquis 2002) or syntactic fusion in a possibilistic framework (Dubois, Lang, & Prade 1994; Benferhat *et al.* 2002a) which is a real advantage at a computational point of view. However these frameworks do not allow for the reversibility of the fusion operations. On a theoretical point of view, reversibility is interesting because it involves the definition of a new framework that enables to express priorities independantly from the merging operators. When facing real scale applications, large amount of data are produced by numerous users. Robust merging techniques and error recovering techniques are necessary. Data management applications require the reversibility of the merging process in case of errors. In archaeological applications, various kinds of errors linked to the measure process may occur. Besides, several surveys of a same object, made at two different instants by a same person or by two different persons may lead to inconsistencies. Indeed, the result of the fusion in a first survey is performed from measures and hypothesis on the object stemming from archeologists' expert knowledge. In the following surveys, new measures may conflict with the hypothesis of the previous survey. Excavations generally take place during several years, surveys made at a certain year may produce knowledge that may invalidate the hypothesis made years before, therefore there is a necessity to come back to initial information. We propose a very general reversible framework for fusion. This framework is suitable for both ordered or not ordered sources as well as for items of information with explicit or implicit priorities or without priorities. Information is represented in propositional calculus and the fusion operation are semantically and syntactically defined. The reversibility of the fusion operations is obtained by an appropriate encoding of the pre-orders on interpretations and on formulas by polynomials on real numbers (Papini 2001; Benferhat *et al.* 2002b).

Preliminaries and notations

In this paper we use propositional calculus, denoted by $\mathcal{L}_{\mathcal{PC}}$, as knowledge representation language with the usual connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$. The lower case letters a, b, c, $\cdots$, are used to denote propositional variables, the lower case

Greek letters ϕ, ψ, $\cdots$, are used to denote formulas, the upper case letters A, B, C, are used to denote sets of formulas. We denote by $\mathcal{W}$ the set of interpretations[1] and by $Mod(\psi)$ the set of models of ψ, that is $Mod(\psi) = \{\omega \in \mathcal{W}, \omega \models \psi\}$ where $\models$ denotes the inference relation used for drawing conclusions. Let ψ and ϕ be formulas and X be a set of formulas, $\psi \models \phi$ denotes that $Mod(\psi) \subseteq Mod(\phi)$ and $X \models \phi$ denotes that $\forall \psi \in X$, $Mod(\psi) \subseteq Mod(\phi)$. The symbol $\equiv$ denotes logical equivalence, and $\psi \equiv \phi$ means that $\psi \models \phi$ and $\phi \models \psi$.

Pre-orders and polynomials

The aim of this section is to briefly recall some definitions on polynomials and to remind how polynomials can be used to represent total pre-orders as well as changes on total pre-orders(Papini 2001)(Benferhat *et al.* 2002b).

Polynomials and pre-order on polynomials Let $\mathbb{R}$ be the set of real numbers. We denote by $\mathbb{R}[x]$ the set of polynomials such that $p = \sum_{i=0}^{n} p_i\, x^i,\ p_i \in \mathbb{R}$. We call right (resp. left) shift of k positions a multiplication (resp. division) by x^k. The support of a polynomial p is the set of elements of $\mathbb{N}$, denoted by S_p, composed by the indices i for which $p_i \neq 0$. Moreover, $max(S_p) = deg(p)$, and $max(\emptyset) = 0$, $deg(p)$ denotes the degree of p.

Pre-orders on polynomials Let p and q be two polynomials on real numbers such that $p = \Sigma_{i=0}^{k}\ p_i\ x^i$ and $q = \Sigma_{i=0}^{l}\ q_i\ x^i$. We use various pre-orders for comparing polynomials.

Maximum The pre-order $\leq_{MAX}$ is:
$p \leq_{MAX} q$ iff $max_{i=0}^{k}(p_i) \leq max_{i=0}^{l}(q_i)$.
Where $max_{i=0}^{l}(q_i)$ denotes the maximum of the set $\{q_0, \cdots, q_l\}$

Sum The pre-order $\leq_{SUM}$ is:
$p \leq_{SUM} q$ iff $\Sigma_{i=0}^{k} p_i \leq \Sigma_{i=0}^{l} q_i$.

Weighted sum Let $\{a_i,\ 1 \leq i \leq k\}$ and $\{b_j,\ 1 \leq j \leq l\}$ two sets of scalars. The pre-order $\leq_{WS}$ is:
$p \leq_{WS} q$ iff $\Sigma_{i=0}^{k} a_i \times p_i \leq \Sigma_{j=0}^{l} b_j \times q_j$.

Lexicographic The pre-order $\leq_{LEX}$ is:
$p \leq_{LEX} q$ iff $\exists i \in \mathbb{N}\ \forall j \in \mathbb{N},\ j < i,\ (p_j = q_j$ and $p_i < q_i)$.

Leximax Let v and w be two vectors composed by the coefficients of p and q ordered in increasing order. Let $p' = \Sigma_{i=0}^{n} v_i\ x^i$ and $q' = \Sigma_{j=0}^{m} w_j\ x^j$ be two polynomials built with the components of v and w respectively. The pre-order $\leq_{LMAX}$ is such that $p \leq_{LMAX} q$ iff $p' \leq_{LEX} q'$.

[1] Interpretations are represented by set of literals.

Representing pre-orders by polynomials Let $(A,\ \leq_A)$ be a finite set with a total pre-order. Representing $\leq_A$ by polynomials requires the definition of a weighting function that assigns each element a_i of A a polynomial. This weighting function is such that $rk(a_i) \in \mathbb{N}$ is the rank of a_i in the pre-order $\leq_A$ [2]. From the binary decomposition of $rk(a_i)$, denoted by $(v_0, \ldots, v_m)$, with $2^{m-1} \leq rk(a_i) < 2^m$, we build a polynomial $p(a_i)$ such that $p(a_i) = \Sigma_{i=0}^{m} v_{m-i}\ x^i$. These polynomials are ordered according to the lexicographic order to represent $\leq_A$. For details see (Papini 2001).

Semantic approach

From a semantic point of view, the priorities between interpretations are represented by polynomials as well as the result of the merging by classical fusion operators. Let $E = \{K_1,\ \ldots,\ K_n\}$ be a set of n propositional bases representing the information provided by n sources . We use two kinds of total pre-orders, a pre-order between the bases, called external pre-order and pre-orders on the interpretations of $\mathcal{L}_{PC}$ relative to each base, called internal pre-orders. In the reversible framework, external and internal pre-orders are total pre-orders represented by polynomials. In the following, preferred elements are minimal elements in total the pre-order.

External pre-order Let $E = \{K_1,\ \ldots,\ K_n\}$ be a set of propositional bases, an external weighting function is a function q that assigns each base K_i an integer called external weight and denoted by $q(K_i)$. An external pre-order denoted by $\leq_E$ is defined such that:

$$\forall K_i,\ K_j \in E,\ \ K_i \leq_E K_j \quad \text{iff} \quad q(K_i) \leq q(K_j)$$

where $q(K_i) = rk(K_i)$. When the sources are explicitly ordered, the weights $q(K_i)$ are the ranks within the total pre-order $\leq_E$. When the sources are not ordered, the bases are equally preferred and $\forall K_i \in E,\ q(K_i) = 0$.

Internal pre-order Let $K_i \in E$ be a propositional bases and $\mathcal{W}$ be the set of interpretations of $\mathcal{L}_{PC}$. An internal weighting function assigns each interpretation ω a polynomial on real numbers called internal weight and denoted by $p_{K_i}(\omega)$. For each base K_i, an internal pre-order denoted by $\leq_{K_i}$ is defined such that:

$$\forall \omega_j,\ \omega_k \in \mathcal{W},\ \ \omega_j \leq_{K_i} \omega_k \ \text{iff}\ p_{K_i}(\omega_j) \leq p_{K_i}(\omega_k)$$

Three cases arise. When a total pre-order is given for K_i, the $p_{K_i}(\omega)$ are encoded by polynomials as mentioned in the polynomial pre-order representation section.[3]. When K_i is implicitely pre-ordered the $p_{K_i}(\omega)$ can be computed using, for example the Hamming distance (see section 6) and encoded by constant polynomials (or integers). Finally, when

[2] We call rank of a_i in the pre-order $\leq_A$ the index of a_i in the list of the elements of A ordered in ascending ordering according to $\leq_A$.

[3] For the sake of homogeneity, since pre-orders are represented by polynomials, weights are encoded by polynomials which reflect the rank of the interpretations in the total pre-order.

no pre-order is defined all the interpretations are equally preferred and we have $\forall \omega \in \mathcal{W},\ p_{K_i}(\omega) = 0$.

Global weight computation

For the semantic approach in the reversible framework, external and internal pre-orders are represented by polynomials. The merging is the combination of these pre-orders in a global pre-order. This is done by the combination of external and internal weights in a global weight independant of the merging operator.

Definition 1 *Let $q(K_i)$ be the external weight for the bases K_i, $1 \leq i \leq n$. The global external weight is such that:*

$$q_{\oplus} = \Sigma_{j=0}^{n-1}\, q(K_{j+1})x^j$$

However, the bases cannot be identified by their rank. It is necessary to define an absolute ranking in order to define an inversible function. An absolute ranking defines a one to one correspondence between ranks and bases. The absolute ranking function is only used to encode internal pre-orders in the global pre-order. The absolute ranking is not a merging priority definition.

Definition 2 *Let $E = \{K_1, \ldots, K_n\}$ be a set of propositional bases. An absolute ranking funtion, denoted by r, is an application from E to $\mathbb{N}$ which assigns each base K_i an absolute rank $r(K_i)$ such that:*

- *if $K_i <_E K_j$ then $r(K_i) < r(K_j)$.*
- *else if $K_i =_E K_j$ and $i < j$ then $r(K_i) < r(K_j)$.*

The Global weight of an interpretation is the sum of all the internal weights shifted as many times as necessary in order to produce disjoint supports. The aim of the disjoint supports is to ensure that each coefficient of an internal weight is not summed with a coefficient from another internal weight. The number of shifts depends on the absolute rank of a base. More formally, for a set $E = \{K_1, \ldots, K_n\}$ of propositional bases, with internal weights $p_{k_i}(\omega)$, $1 \leq i \leq n$ and external weights $q(K_i)$. Let r be the absolute ranking funtion for E. The global weight for an interpretation ω, denoted by $p_{K_1 \oplus \cdots \oplus K_n}(\omega)$ is such that

$$p_{K_1 \oplus \cdots \oplus K_n}(\omega) = \Sigma_{i=1}^{n}\, p_{K_i}(\omega)\, x^{\Sigma_{j=1}^{r(K_i)-1} MAX_{r^{-1}(j)}}$$

with $MAX_{r^{-1}(j)} = \max_{\omega' \in \mathcal{W}} deg(p_{r^{-1}(j)}(\omega')) + 1$. In the following for the sake of simplicity, we denote by $p_{\oplus}(\omega)$ the global weight.

Semantic merging within the reversible framework

In the semantic approach, merging expresses a global pre-order on the global weights. The result is the set of preferred interpretations in this pre-order. Within the reversible framework, the global pre-order, denoted by $\leq_{K_1 \oplus \cdots \oplus K_n}$ is such that $\forall \omega,\ \omega' \in \mathcal{W},\ \omega \leq_{K_1 \oplus \cdots \oplus K_n} \omega'$ iff $p_{\oplus}(\omega) \leq p_{\oplus}(\omega')$. The choice of the merging operator provides a way for comparing global weight polynomials. The use of $\leq_{MAX}$ provides the behavior of the MAX merging operator, the use of $\leq_{SUM}$ provides the behavior of the SUM merging operator, and so on. The following example illustrates the reversible framework for merging.

Example 1 *Let's use the well known example given in (Revesz 1993). Let $K_1 = \{(s \vee o) \wedge \neg d\}$, $K_2 = \{(\neg s \wedge d \wedge \neg o) \vee (\neg s \wedge \neg d \wedge o)\}$ and $K_3 = \{s \wedge o \wedge d\}$ three propositional bases. The set of interpretations $\mathcal{W}$ is sucht that $\omega_0 = \{\neg s, \neg d, \neg o\}, \omega_1 = \{\neg s, \neg d, o\}, \omega_2 = \{\neg s, d, \neg o\}, \omega_3 = \{\neg s, d, o\}, \omega_4 = \{s, \neg d, \neg o\}, \omega_5 = \{s, \neg d, o\}, \omega_6 = \{s, d, \neg o\}, \omega_7 = \{s, d, o\}$. The external pre-order is $K_3 \leq_E K_1 =_E K_2$ thus the external weights are $q(K_1) = 2$, $q(K_2) = 2$, $q(K_3) = 1$. The computation of external global weight gives $q_{\oplus} = 2 + 2x + x^2$. The absolute ranking function is defined by $r(K_1) = 2$, $r(K_2) = 3$, $r(K_3) = 1$ and $r^{-1}(1) = K_3$, $r^{-1}(2) = K_1$, $r^{-1}(3) = K_2$. Table 1 shows the result of the computation of internal and global weights. We have $MAX_{r^{-1}(1)} = MAX_{K_3} = max_{\omega' \in \mathcal{W}}\, deg(p_{K_3}(\omega')) + 1 = 2$, $MAX_{r^{-1}(2)} = MAX_{K_1} = 2$ and $MAX_{r^{-1}(3)} = MAX_{K_2} = 3$. For an interpretation ω_i the global weight is $p_{\oplus}(\omega_i) = p_{K_3}(\omega_i) + p_{K_1}(\omega_i)x^2 + p_{K_2}(\omega_i)x^4$. If we use the SUM merging operator, the pre-order $\leq_{SUM}$ is used and the global pre-order is $\omega_1 =_{SUM} \omega_2 =_{SUM} \omega_5 =_{SUM} \omega_7 <_{SUM} \omega_3 =_{SUM} \omega_4 =_{SUM} \omega_6 <_{SUM} \omega_0$. Minimal interpretations in this pre-order are the set $Mod(K_1 \oplus \cdots \oplus K_n) = \{\omega_1, \omega_2, \omega_5, \omega_7\}$.*

ω	p_{K_1}	p_{K_2}	p_{K_3}	$p_{\oplus}$
ω_0	x	1	$1+x$	$1+x+x^3+x^4$
ω_1	1	0	x	$x+x^2$
ω_2	x	0	x	$x+x^3$
ω_3	x	1	1	$1+x^3+x^4$
ω_4	1	x^2	x	$x+x^2+x^6$
ω_5	0	1	1	$1+x^4$
ω_6	x	1	1	$1+x^3+x^4$
ω_7	x	x^2	0	x^3+x^6

Table 1: *Interprétations, internal and global weights*

ω	$\leq_{MAX}$	$\leq_{SUM}$	$\leq_{WS}$	$\leq_{LEX}$	$\leq_{LMAX}$
ω_0	1	4	6	5	5
ω_1	1	2	3	1	1
ω_2	1	2	3	2	2
ω_3	1	3	5	4	4
ω_4	1	3	5	6	6
ω_5	1	2	3	3	3
ω_6	1	3	5	4	4
ω_7	1	2	4	7	7

Table 2: *Interprétations and rank for differents merging operators*

Reversibility

Reversibility allows us to retrieve the external pre-order as well as internals pre-orders from the global weight. Let $q_{\oplus}$ be the external weight polynomial, by the construction of the polynomial, the number of propositional bases is $n = deg(q_{\oplus}) + 1$. Moreover from the polynomial encoding of the global external weight, it is possible to retrieve all the externals weights

$$q(K_i) = \frac{q_{\oplus}\ mod\ x^i}{x^{i-1}}$$

and therefore the absolute ranks. Polynomials also allow us to retrieve the internal weights from global weights assigned to interpretations. Since the construction of global weights right shifts the internal weights in order to produce disjunct supports, the inverse operation consists in breaking the global weight into internal weights by left shifting a number of times equal to the maximum degree of the support corresponding to the greatest internal weight of the base. More formally, for each interpretation ω and for each propositional base K_i we have:

$$p_{K_i}(\omega) = \frac{p_{\oplus}(\omega)\ mod\ x^{\Sigma_{l=1}^{r(K_i)} MAX_{r^{-1}(l)}}}{x^{\Sigma_{k=1}^{r(K_i)-1} MAX_{r^{-1}(k)}}}$$

Example 2 *Using the results of example 1, we illustrate the reversibility. From the polynomial $q_{\oplus} = 2 + 2x + 1x^2$ we can retreive the number of merged bases that is $deg(q_{\oplus}) + 1 = 3$. Moreover, we can retreive the external weights since $q(K_1) = q_{\oplus}\ mod\ x^1 = 2$, $q(K_2) = \frac{q_{\oplus}\ mod\ x^2}{x} = 2$, and $q(K_3) = \frac{q_{\oplus}\ mod\ x^3}{x^2} = 1$ and thus $K_3 \leq_E K_1 =_E K_2$. The external weights allows us to recover the absolute ranks $r(K_1) = 2$, $r(K_2) = 3$, $r(K_3) = 1$ and $r^{-1}(1) = K_3$, $r^{-1}(2) = K_1$, $r^{-1}(3) = K_2$. We can retrieve the internal weights as follows. With $MAX_{r^{-1}(1)} = MAX_{K_3} = 2$, $MAX_{r^{-1}(2)} = MAX_{K_1} = 2$ and $MAX_{r^{-1}(3)} = MAX_{K_2} = 3$. For example, let $p_{\oplus}(\omega_3) = 1 + x^3 + x^4$ be the global weight for the interpretation ω_3. The internal weights are the followings :*

$$p_{K_1}(\omega) = \frac{p_{\oplus}(\omega)\ mod\ x^{\Sigma_{l=1}^{r(K_1)} MAX_{r^{-1}(l)}}}{x^{\Sigma_{k=1}^{r(K_1)-1} MAX_{r^{-1}(k)}}} = \frac{p_{\oplus}(\omega)\ mod\ x^{MAX_{K_3}+MAX_{K_1}}}{x^{MAX_{K_3}}} = \frac{1+x^3+x^4\ mod\ x^4}{x^2} = x$$

then $p_{K_2}(\omega) = \frac{1+x^3+x^4\ mod\ x^7}{x^4} = 1$ and $p_{K_3}(\omega) = 1 + x^3 + x^4\ mod\ x^2 = 1$.

Syntactic approach

Let $\mathcal{B} = \{\Sigma_1, \ldots, \Sigma_n\}$ be a set of n weighted bases. Each base Σ_i is a finite set of weighted formulas such that $\Sigma_i = \{(\phi_j, p_{\Sigma_i}(\phi_j)) \mid \phi_j \in \mathcal{L_{PC}}, p_{\Sigma_i}(\phi_j) \in \mathbb{R}[x]\}$. In the reversible framework, external and internal preferences are respectively represented by an external total pre-order on the weighted bases and by internal total pre-orders on the formulas. These total pre-orders are encoded by polynomials. In the syntactic approach, preferred items of the total pre-orders on the formulas are the maximum in the pre-orders.

External pre-order Let $\mathcal{B} = \{\Sigma_1, \ldots, \Sigma_n\}$ be a set of weighted bases, an external weighting function is a function that assigns each weighted base an integer denoted by $q(K_i)$. An external pre-order denoted by $\leq_{\mathcal{B}}$ is defined such that:

$$\forall \Sigma_i, \Sigma_j \in \mathcal{B},\ \Sigma_i \leq_{\mathcal{B}} \Sigma_j \text{ iff } q(\Sigma_i) \leq q(\Sigma_j)$$

where $q(K_i) = rk(\Sigma_i)$. When the sources are explicitly ordered, the weights $q(\Sigma_i)$ are the ranks within the total pre-order $\leq_{\mathcal{B}}$. When the sources are not ordered, the bases are equally preferred and $\forall \Sigma_i \in \mathcal{B}, q(\Sigma_i) = 0$.

Internal pre-order Let $\Sigma_i \in \mathcal{B}$ be a weighted base. An internal weighting function for Σ_i assigns each formula ϕ of Σ_i a polynomial on real numbers denoted by $p_{\Sigma_i}(\phi)$. An internal pre-order denoted by $\leq_{\Sigma_i}$ is defined such that:

$$\forall \phi, \psi \in \Sigma_i,\ \phi \leq_{\Sigma_i} \psi \text{ iff } p_{\Sigma_i}(\phi) \leq p_{\Sigma_i}(\psi)$$

Three cases arise. When a total pre-order is given for the Σ_i, the $p_{\Sigma_i}(\phi)$ are encoded by polynomials as mentioned in polynomial pre-orders section. When Σ_i is implicitly pre-ordered the $p_{\Sigma_i}(\phi)$ can be computed as constant polynomials. Finally, when no pre-order is defined all the formulas are equally preferred and we have $\forall \phi \in \Sigma_i, p_{\Sigma_i}(\phi) = 0$.

Computation of the global weighted base

The merging of weighted bases is the construction of a base containing the formulas of each base, the disjunctions of two formulas coming from two bases, the disjunctions of three formulas coming from three bases, and so on until disjunctions of n formulas coming from n bases. For that, a global weight has to be computed.

Definition 3 *Let $q(\Sigma_i)$ be the external weight for Σ_i. The global external weight is such that:*

$$q_{\otimes} = \Sigma_{j=0}^{n-1} q(\Sigma_{j+1})\, x^j$$

In the syntactic approach for merging, global weighted base is composed by formulas with a global weight. This weight has to take into account the external pre-order. However, the bases cannot be identified by their rank. It is necessary to define an absolute ranking function in order to define inversible function.

Definition 4 *Let $\mathcal{B} = \{\Sigma_1, \ldots, \Sigma_n\}$ be a set of weighted bases. An absolute ranking function, denoted by r, is an application from $\mathcal{B}$ to $\mathbb{N}$ which assigns each base Σ_i an absolute rank $r(\Sigma_i)$ such that:*

- *if $\Sigma_i <_{\mathcal{B}} \Sigma_j$ then $r(\Sigma_i) < r(\Sigma_j)$*
- *else if $\Sigma_i =_{\mathcal{B}} \Sigma_j$ and $i < j$ then $r(\Sigma_i) < r(\Sigma_j)$*

The construction of the global weighted base requires the definition of the disjunction of k formulas, denoted by D_k. The disjunction is such that:

$$D_k = \phi_{j_1} \vee \cdots \vee \phi_{j_i} \vee \cdots \vee \phi_{j_k}$$

where each ϕ_{j_i} comes from a different base Σ_i. Moreover, we denote by s the mapping which assigns each formula of D_k the weighted base from where it is coming from. More formally, let $D_k = \phi_{j_1} \vee \cdots \vee \phi_{j_i} \vee \cdots \vee \phi_{j_k}$, if $(\phi_{ji}, p_{\Sigma_l}(\phi_{ji})) \in \Sigma_l$, then $s(\phi_{ji}) = \Sigma_l$. The definition of D_k and s allows us to define a global weight.

Definition 5 *Let $D_k = \phi_{j_1} \vee \cdots \vee \phi_{j_i} \vee \cdots \vee \phi_{j_k}$ be a disjunction of formulas coming from k weighted bases. The global weight of D_k, denoted by $p_{\Sigma_1 \otimes \cdots \otimes \Sigma_n}(D_k)$ is such that:*

$$p_{\Sigma_1 \otimes \cdots \otimes \Sigma_n}(D_k) = \sum_{i=1}^{k} p_{s(\phi_{ji})}(\phi_{ji}) \times x^{\Sigma_{m=1}^{r(s(\phi_{ji}))-1} MAX_{r^{-1}(m)}}$$

with $MAX_{r^{-1}(m)} = max_{\phi' \in r^{-1}(m)}(deg(p_{r^{-1}(m)}(\phi')) + 1)$.

For the sake of simplicity, $p_{\otimes}(D_k)$ denotes $p_{\Sigma_1 \otimes \cdots \otimes \Sigma_n}(D_k)$. The global weighted base consists in all the possible disjunctions of formulas from the bases of $\mathcal{B}$ assigned a global weight. More formally:

Definition 6 *Let $\mathcal{B} = \{\Sigma_1, \ldots, \Sigma_n\}$ be a set of weighted bases. Let D_k be a disjunction of k formulas. The global weighted base, denoted by Σ_g is such that:*

$$\Sigma_g = \bigcup_{k=1}^{n} \{(D_k, p_{\otimes}(D_k))\}$$

Syntactic merging and reversible framework

The result of the merging process in the syntactical approach is the set of weighted formulas of maximal global weights according to the pre-order defined as follows. Let $(\phi, p_{\otimes}(\phi)),\ (\psi, p_{\otimes}(\psi)) \in \Sigma_g$ two weighted formulas of the global stratified base. The global pre-order, denoted by $\leq_{\Sigma_1 \otimes \cdots \otimes \Sigma_n}$ is such that:

$$\phi \leq_{\Sigma_1 \otimes \cdots \otimes \Sigma_n} \psi \text{ iff } p_{\otimes}(\phi) \leq p_{\otimes}(\psi)$$

The choice of a merging operator involves the use of a specific pre-order on polynomials. For example, the use of the MAX operator involves the use of the $\leq_{MAX}$ pre-order. The following example illustrates the syntactic merging.

Example 3 *Let $\mathcal{B} = \{\Sigma_1, \Sigma_2, \Sigma_3\}$ be a set of weighted bases such that $\Sigma_1 = \{(\phi_1, 1)\}$, $\Sigma_2 = \{(\phi_2, x^2)\}$ and $\Sigma_3 = \{(\phi_3, 1+x)\}$. We give an arbitrary external pre-order pre-order is such that $\Sigma_3 \leq_{\mathcal{B}} \Sigma_1 =_{\mathcal{B}} \Sigma_2$, thus the external weights are $q(\Sigma_1) = 2$, $q(\Sigma_2) = 2$ and $q(\Sigma_3) = 1$. The computation of external global weight gives*

$$\begin{aligned} q_{\otimes} &= q(\Sigma_1) \times x^0 + q(\Sigma_2) \times x^1 + q(\Sigma_3) \times x^2 \\ &= q(\Sigma_1) + q(\Sigma_2)x + q(\Sigma_3)x^2 \\ &= 2 + 2x + x^2 \end{aligned}$$

The absolute ranking function is defi ned by $r(\Sigma_1) = 2$, $r(\Sigma_2) = 3$, $r(\Sigma_3) = 1$ and $r^{-1}(1) = \Sigma_3$, $r^{-1}(2) = \Sigma_1$, $r^{-1}(3) = \Sigma_2$. We have $MAX_{r^{-1}(1)} = MAX_{\Sigma_3} = max_{\phi' \in \Sigma_3} deg(p_{\Sigma_3}(\phi')) + 1 = 2$, $MAX_{r^{-1}(2)} = MAX_{\Sigma_1} = 1$ and $MAX_{r^{-1}(3)} = MAX_{\Sigma_2} = 3$. Moreover $s(\phi_1) = \Sigma_1$, $s(\phi_2) = \Sigma_2$ and $s(\phi_3) = \Sigma_3$. According to the defi nition, $(\phi_i, p_{\otimes}(\phi_i)) \in \Sigma_g$ and the global internal weights $p_{\otimes}(\phi_i)$ are computed as follows:

$$\begin{aligned} p_{\otimes}(\phi_1) &= \textstyle\sum_{i=1}^{k} p_{s(\phi_1)}(\phi_1) \times x^{\sum_{m=1}^{r(s(\phi_1))-1} MAX_{r^{-1}(m)}} \\ &= \textstyle\sum_{i=1}^{k} p_{\Sigma_1}(\phi_1) \times x^{\sum_{m=1}^{r(\Sigma_1)-1} MAX_{r^{-1}(m)}} \\ &= p_{\Sigma_1}(\phi_1) \times x^{MAX_{r^{-1}(1)}} \\ &= p_{\Sigma_1}(\phi_1) \times x^{MAX_{\Sigma_3}} \\ &= p_{\Sigma_1}(\phi_1) \times x^2 = x^2 \end{aligned}$$

Using the same way we have $p_{\otimes}(\phi_2) = x^5$ and $p_{\otimes}(\phi_3) = 1 + x$. For disjunctions of more than one formula, the computation is recursive. As we can see:

$$\begin{aligned} p_{\otimes}(\phi_1 \vee \phi_3) &= p_{\Sigma_1}(\phi_1) \times x^{MAX_{\Sigma_3}} + p_{\Sigma_3}(\phi_3) \times x^0 \\ &= p_{\otimes}(\phi_1) + p_{\otimes}(\phi_3) \\ &= x^2 + x + 1 \end{aligned}$$

The computation of all the disjunctions D_k and all the global weights gives the global weighted base:

$$\begin{aligned} \Sigma_g = \{ & (\phi_1, x^2), (\phi_2, x^5), (\phi_3, 1+x), \\ & (\phi_1 \vee \phi_2, x^5 + x^2), (\phi_1 \vee \phi_3, x^2 + x + 1), \\ & (\phi_2 \vee \phi_3, x^5 + x + 1), \\ & (\phi_1 \vee \phi_2 \vee \phi_3, x^5 + x^2 + x + 1)\} \end{aligned}$$

A global pre-order characterizes the behavior of the merging operator, for example, if the operator SUM is used, the result of the merging is $\Sigma_1 \otimes \Sigma_2 \otimes \Sigma_3 = \{(\phi_1 \vee \phi_2 \vee \phi_3, x^5 + x^2 + x + 1)\}$.

Reversibility

The reversibility allows us to retrieve the external and internal pre-orders from the global weight. Let $q_{\otimes}$ be the external weight polynomial, the number of propositional bases is $n = deg(q_{\otimes}) + 1$. Moreover, it is possible to retrieve external weights:

$$q(\Sigma_i) = \frac{q_{\otimes} \ mod \ x^i}{x^{i-1}}$$

and therefore the absolute ranks. Polynomials also allow us to retrieve internal weights assigned to the formulas. Since the construction of global weight rights shift the internal weights in order to produce disjunct supports, the inverse operation consists in breaking the global weight into internal weights by left shifting a number of times equal to the maximum degree of the support corresponding to the greatest internal weight of the base. More formally, let $\psi = \phi_{j_1} \vee \cdots \vee \phi_{j_i} \vee \cdots \vee \phi_{j_n}$ be a formula coming from the disjunction of n formulas, that is a formula wich weight is composed of a maximum number of monoms, we have

$$p_{\Sigma_i}(\phi_j) = \frac{p_{\otimes}(\psi) \ mod \ x^{\Sigma_{l=1}^{r(s(\phi_j))} MAX_{r^{-1}(l)}}}{x^{\Sigma_{k=1}^{r(s(\phi_j))-1} MAX_{r^{-1}(k)}}}$$

Example 4 *Coming back to the previous example, $(\phi_1 \vee \phi_2 \vee \phi_3, x^5 + x^2 + x + 1)$ is the formula coming from Σ_g with weight composed of a maximum number of monoms. The polynomial $q_{\otimes} = 2 + 2x + x^2$ allows us to know that three bases have been merged because $deg(q_{\otimes}) + 1 = 3$. Moreover, $q(\Sigma_1) = 2$, $q(\Sigma_2) = 2$, $q(\Sigma_3) = 1$. The external weights enable to recover the function r with $r(\Sigma_1) = 2$, $r(\Sigma 2) = 3$, $r(\Sigma 3) = 1$ and therefore $r^{-1}(1) = \Sigma_3$, $r^{-1}(2) = \Sigma_1$, $r^{-1}(3) = \Sigma_2$. Moreover $s(\phi_1) = \Sigma_1$, $s(\phi_2) = \Sigma_2$ and $s(\phi_3) = \Sigma_3$ The internal weights are $p_{\Sigma_1}(\phi_1) = \frac{p_{\otimes}(\psi) \ mod \ x^{\Sigma_{l=1}^{r(s(\phi_1))} MAX_{r^{-1}(l)}}}{x^{\Sigma_{k=1}^{r(s(\phi_1))-1} MAX_{r^{-1}(k)}}} = \frac{x^5+x^2+x+1 \ mod \ x^3}{x^2} = 1$, $p_{\Sigma_2}(\phi_2) = \frac{x^5+x^2+x+1 \ mod \ x^6}{x^3} = x^2$, and $p_{\Sigma_3}(\phi_3) = \frac{x^5+x^2+x+1 \ mod \ x^2}{x^0} = 1 + x$.*

Equivalence between semantic and syntactic approaches

We now show the equivalence between the semantic and syntactic approaches in the reversible framework. For that, like in the possibilistic framework or in the system Z (Benferhat *et al.* 2002a) (Pearl 2003) we use a function denoted by κ_{Σ_i} which for each weighted base Σ_i attaches to each interpretation ω the maximal weight of the formulas of Σ_i falsified by ω. More formally, $\forall \omega \in \mathcal{W}$, $\kappa_{\Sigma_i}(\omega) = max(\{p_{\Sigma_i}(\phi), (\phi, p_{\Sigma_i}(\phi)) \in \Sigma_i \ and \ \omega \not\models \phi\})$. This function allows us to define the syntactic counterpart of a propositional base.

Definition 7 *Let $E = \{K_1, \ldots, K_n\}$ be a set of propositional bases and let $\mathcal{B} = \{\Sigma_1, \ldots, \Sigma_n\}$ be a set of* *weighted bases, Σ_i is syntactic counterpart of K_i iff $\forall \omega \in \mathcal{W}$, $\kappa_{\Sigma_i}(\omega) = p_{K_i}(\omega)$. Moreover, $\mathcal{B}$ is syntactic counterpart of E if and only if each weighted base of Σ_i de $\mathcal{B}$ is a syntactic counterpart of $K_i \in E$ and $q(K_i) = q(\Sigma_i)$ (the external pre-orders are the same).*

Equivalence between the two approaches is provided by the construction of the syntactic counterpart of a set of propositional bases. For that, we construct the syntactic counterpart of a propositional base as follows. For each interpretation ω ranked according to the internal pre-orders. We first generate all the formulas falsified by ω for K_i and attach to them the internal weight $p_{K_i}(\omega)$. We then remove the formulas falsified by an interpretation already processed. We finally remove subsummed formulas [4] and discard the formulas with null weight. From the construction of the syntactic counterpart of a set of propositional bases, we show the equivalence between semantic and syntactic approaches for merging within in the reversible framework.

Algorithm 1 syntactic counterpart

$\Sigma \leftarrow \emptyset, M \leftarrow \emptyset, S \leftarrow \emptyset, T \leftarrow \emptyset$
for each $\omega \in \mathcal{W}$ **do**
  $S \leftarrow \emptyset, T \leftarrow \emptyset$
  $\Sigma' \leftarrow \{(D_j, p_{K_i}(\omega)),\ 1 \le j \le card(\omega),\ \omega \not\models D_j)\}$
  $M \leftarrow M \cup \Sigma'$
  $S \leftarrow \{(D_j, p_{K_i}(\omega)) \in \Sigma',\ \exists (D_j, p_{K_i}(\omega')) \in M,\ with\ p_{K_i}(\omega') < p_{K_i}(\omega)\}$
  $\Sigma' \leftarrow \Sigma' - S$
  $T \leftarrow \{(D_k, p_{K_i}(\omega)) \in \Sigma' \mid \exists (D_j, p_{K_i}(\omega)) \in \Sigma',\ D_k \models D_j\}$
  $\Sigma' \leftarrow \Sigma' - T$
  $\Sigma \leftarrow \Sigma \cup \Sigma'$
end for
$\Sigma \leftarrow \{(\phi,\ p_{\Sigma_i}(\phi)) \in \Sigma_i \mid p_{\Sigma_i}(\phi) \neq 0\}$
return Σ

Proposition 1 *Let $E = \{K_1, \ldots K_n\}$ be a set of propositional bases and let $\mathcal{B} = \{\Sigma_1, \ldots \Sigma_n\}$ be its syntactic counterpart.*

$$\forall \omega \in \mathcal{W},\ \kappa_{\Sigma_1 \otimes \cdots \otimes \Sigma_n}(\omega) = p_{K_1 \oplus \cdots \oplus K_n}(\omega)$$

The proof is based on the construction of the global pre-order according to the semantic approach and the global weighted base according to the syntactic approach. We shows that the global weights are the same in the two approaches.

Generalizations

The approach for merging propositional belief bases stemming from Hamming distance (S. & R. 2005) can be easily captured within our reversible framework. In this approach no pre-order between sources is considered. In contrast local pre-orders are implicit pre-orders induced by the Hamming distance between interpretations. A distance between an interpretation ω and a propositional belief base K_i is defined by $d(\omega,\ K_i) = min_{\omega' \in Mod(K_i)}(d(\omega,\ \omega'))$. In our framework, for each belief base K_i an internal weight is a constant polynomial such that $p_{K_i}(\omega) = d(\omega,\ K_i)$. Performing the merging of n propositional belief bases amounts to compute a global weight as presented, the distance based fusion operators are represented within the reversible framework by pre-orders on polynomials. The pre-orders $\le_{MAX}$, $\le_{SUM}$, $\le_{WS}$ and $\le_{LMAX}$ are used to compare the polynomials corresponding to the global weights for the fusion operators MAX, SUM, WS et $LMAX$ given in (Konieczny & Pérez 1998). Like in (Benferhat *et al.* 2002a), the proposed framework gives a syntactic counterpart to the distance based fusion operations and moreover brings reversibility to both semantic and syntactic approaches. Besides the proposed reversible framework allows us to generalize the revision of an epistemic state by another epistemic state to the fusion of several epistemic states. Let $\Psi_1, \ldots, \Psi_n$, be n epistemic states. each epistemic state Ψ_i can be represented by a total pre-order on interpretations $\le_{\Psi_i}$ or by a weighted belief base Σ_i. In case of revision $n = 2$. Revision can be easily captured within the proposed framework. According to a semantical point of view, the two epistemic states Ψ_1 and Ψ_2 are respectively represented by the internal pre-orders $\le_{\Psi_1}$ and $\le_{\Psi_2}$. For the revision of Ψ_1 by Ψ_2, the external pre-order is $\Psi_1 <_E \Psi_2$. For the revision with memory proposed in (Papini 2001), the global pre-order is obtained from the lexicographic pre-order on polynomials $\le_{LEX}$. Under these hypothesis, we found again the results presented in (Benferhat *et al.* 2000). According to the syntactic approach, the two epistemic states Ψ_1 and Ψ_2 are respectively represented by the weighted bases Σ_1 and Σ_2. The external pre-order is the same as for the semantic approach and is denoted by $\Psi_1 <_{\mathcal{B}} \Psi_2$. After the construction of the global weight, the global pre-order on formulas is obtained by means of the lexicographic pre-order on polynomials. For $n > 2$, the reversible framework makes it possible to represent the fusion of epistemic states.

Conclusion

In this paper we presented a very general reversible framework for merging propositional belief bases. It makes it possible to represent within the same framework the case where the sources are ordered or not as well as the case where the items of information are explicitely or implicitely ordered or not ordered. We proposed both semantic and syntactic approaches for fusion within the reversible framework and we showed the equivalence between the semantic and syntactic approaches. We sowed that the proposed framework allows us to represent with a reversible framework the approach of merging propositional belief bases with implicit priorities stemming from Hamming distance and to provide a syntactic counterpart. We also showed that this framework allows for generalizing the revision of an epistemic state by another epistemic state to fusion of epistemic states in the case where the epistemic states are represented by total pre-orders.

In the context of submarine archeology, the construction of models of archeological objects requires photogrammetric measures and measures in laboratory. These measures are represented in propositional calculus and the proposed

[4] $(\phi,\ p_{\Sigma_i}(\phi))$ is subsummed by $(\psi,\ p_{\Sigma_i}(\psi))$ iff $\psi \models \phi$ and $p_{\Sigma_i}(\psi)) \le p_{\Sigma_i}(\phi)$

reversible framework is suitable for the fusion of such pieces of information. However, we also have to deal with structured , semi-structured or hierarchical pieces of information. The fusion of such items of information is still problematical and will be the focus of a future work.

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